

Geometric and Analytic aspects of moduli spaces of Higgs bundles

Abstracts of Talks

Tamás Hausel (June 7, 11:00-11:45)

Explicit Hitchin map on Lagrangians

I will define the notion of equivariant multiplicity algebra of an upward flow in the Higgs moduli space borrowing it from the Arnold school. We can compute it in the stable type $(1, \dots, 1)$ case in terms of the cohomology ring of Grassmannians, and in the wobbly $(1, \dots, 1)$ case we can compute it in the affine Grassmannian. Finally, I will mention a recent computation by Hitchin for examples of multiplicity algebras in the type (2) case for $g=2,3$. Joint project with Nigel Hitchin.

Peter Gothen (June 7, 16:30-17:15)

A general Cayley correspondence for Higgs bundles and higher Teichmüller theory

Let G be a non-compact Lie group of Hermitian type such that the associated Hermitian symmetric space is of tube type. Then the Cayley correspondence for G -Higgs bundles produces special connected components in the moduli space of maximal G -Higgs bundles, which can be seen as moduli spaces of twisted Higgs bundles for a different group. We generalise this correspondence to a larger class of real reductive Lie groups G . Our generalisation encompasses Hitchin components when G is a split real form and special components discovered recently for $SO(p, q)$, and also constructs previously unknown components for the quaternionic real forms of E_6 , E_7 , E_8 and F_4 . The construction is based on a new class of sl_2 -triples in a complex simple Lie algebra, which we call magical. The classification of magical sl_2 -triples is in bijection with the set of theta-positive structures in the sense of Guichard-Wienhard, thus our construction conjecturally detects all examples of higher Teichmüller spaces.

The talk is based on <https://arxiv.org/abs/2101.09377>, which is joint work with Steve Bradlow, Brian Collier, Oscar Garcia-Prada, and André Oliveira.

Daniele Alessandrini (June 7, 17:15-18:00)

The nilpotent cone in rank one and minimal surfaces

I will describe two interesting and closely related moduli spaces: the nilpotent cone in the moduli spaces of Higgs bundles for $SL_2(C)$ and $PSL_2(C)$, and the moduli space of equivariant minimal surfaces in the hyperbolic 3-space. A deep understanding of these objects is important because of their relations with several fundamental constructions in geometry: singular fibers of the Hitchin fibration, branes, mirror symmetry, branched hyperbolic structures, minimal surfaces in hyperbolic 3-manifolds and so on. A stratification of the nilpotent cone is well known and was rediscovered by many people. The closures of the strata are the irreducible components of the nilpotent cone. The talk will focus on describing the intersections between the different irreducible components. This is joint work with Qionglin Li and Andrew Sanders.

Alexander Thomas (June 8, 11:00-11:45)

Holomorphic differentials in flat connections

The non-abelian Hodge correspondence allows to associate to a Higgs bundle a flat connection. We address the following question: What happens if the Higgs field is not supposed to be holomorphic? We show that in this context, the punctual Hilbert scheme of the plane naturally appears. Restricting to the zero-fiber of the Hilbert scheme, we can extract holomorphic differentials out of the flat connection which are not determined by the Higgs field.

Victoria Hoskins (June 8, 11:45-12:30)

Motives of moduli spaces of Higgs bundles

Various computations of cohomological invariants of Higgs bundle moduli spaces should be both unified and refined by working with motivic invariants, which encode finer invariants, like Hodge structures on cohomology groups and also algebro-geometric invariants such as Chow groups. I will give a short introduction to motives and then present some joint work with Simon Pepin Lehalleur on the motive of the Higgs moduli space for coprime invariants. Our geometric strategy starts with Hitchin's scaling action on the Higgs moduli space. First, we use this scaling action together with Harder–Narasimhan recursions and variation of stability for moduli stacks of chains to show the motive of the Higgs moduli space is a direct factor of the motive of a sufficiently large power of the curve. We also use this scaling action to prove a motivic non-abelian Hodge correspondence: the integral Voevodsky motives of the Higgs and de Rham moduli spaces are isomorphic, and thus have isomorphic Chow rings. If time permits, I will also present some explicit formulas in ranks 2 and 3, which is joint also with Lie Fu.

Laura Fredrickson (June 8, 17:15-18:00)

ALG Gravitational Instantons and Hitchin Moduli Spaces

Four-dimensional complete hyperkähler manifolds can be classified into ALE, ALF, ALG, ALG*, ALH, ALH* families. It has been conjectured that every ALG or ALG* hyperkähler metric can be realized as a 4d Hitchin moduli space. I will describe ongoing work with Rafe Mazzeo, Jan Swoboda, and Hartmut Weiss to prove a special case of the conjecture, and some consequences. The hyperkähler metrics on Hitchin moduli spaces are of independent interest, as the physicists Gaiotto—Moore—Neitzke give an intricate conjectural description of their asymptotic geometry.

Olivier Biquard (June 9, 11:00-11:45)

A Milnor-Wood inequality for complex variations of Hodge structure

We study the \mathbb{C}^* -fixed points in spaces of Higgs bundles over a compact Riemann surface for a real semisimple Lie group. We introduce a topological invariant which generalizes the Toledo invariant, and prove an inequality which generalizes both the Milnor-Wood inequality and the Arakelov inequality for variations of Hodge structure. We also establish rigidity results in the maximal case. Joint work with Brian Collier, Oscar Garcia-Prada, and Domingo Toledo.

Qionglin Li (June 9, 11:45-12:30)

Complete solutions of Toda equations and cyclic Higgs bundles over non-compact surfaces

On a Riemann surface with a holomorphic r -differential, one can naturally define a Toda equation and a cyclic Higgs bundle with a grading. A solution of the Toda equation is equivalent to a harmonic metric of the Higgs bundle for which the grading is orthogonal. In this talk, we focus on a general non-compact Riemann surface with an r -differential which is not necessarily meromorphic at infinity. We introduce the notion of complete solution of the Toda equation, and we prove the existence and uniqueness of a complete solution by using techniques for both Toda equations and harmonic bundles. Moreover, we show some quantitative estimates of the complete solution. This is joint work with Takuro Mochizuki (RIMS).

Subhojoy Gupta (June 10, 11:00-11:45)

Dominating $\mathrm{PSL}(n, \mathbb{C})$ -representations of
punctured-surface groups

For a closed and oriented surface S , Derooin-Tholozan had proved that for any representation of the fundamental group of S into $\mathrm{PSL}(2, \mathbb{C})$, there is a dominating Fuchsian representation. They conjectured a generalization in the context of Higgs bundles. I shall describe this context, and talk of the following result for the case when S has punctures: for a generic representation of the punctured-surface group into $\mathrm{PSL}(n, \mathbb{C})$, there is a dominating Hitchin representation in the same relative representation variety. The proof uses Fock-Goncharov coordinates for the moduli space of framed representations.

Steven Bradlow (June 10, 16:30-17:15)

Branched hyperbolic structures from ordinary Higgs
bundles on smooth surfaces

In his famous 1987 paper, in one of the first triumphs for Higgs bundles, Hitchin described the Teichmüller space of a closed surface by using rank two Higgs bundle data to construct smooth hyperbolic structures. Since then related gauge-theoretic approaches using parabolic structures or vortices have extended such constructions to non-smooth hyperbolic structures with conic singularities, thereby shedding new light on results such as the McOwen-Troyanov existence theorem for such metrics with prescribed singularities. In this talk I will describe an approach to singular hyperbolic structures that revisits Hitchin's original construction on ordinary rank two Higgs bundles. This is based on joint work with Indranil Biswas, Sorin Dumitrescu, and Sebastian Heller.

Brian Collier (June 10, 17:15-18:00)

Global Slodowy slices for moduli spaces of
 λ -connections

The moduli spaces of Higgs bundles and holomorphic connections both have important affine holomorphic Lagrangian subvarieties, these are the Hitchin section and the space of opers, respectively. Both of these spaces arise from the same Lie theoretic mechanism, namely a regular nilpotent element of a Lie algebra. In this talk we will generalize these parameterizations to other nilpotents by describing a bundle version of the Slodowy slice through a nilpotent. The resulting objects are not related by the nonabelian Hodge correspondence, but by an operation called the conformal limit. Time permitting, we will also discuss their relation to Higher Teichmüller spaces and maximal variations of Hodge structure.

Adrian Langer (June 11, 11:00-11:45)

Semistable modules over Lie algebroids

Modules over Lie algebroids generalize Higgs bundles and vector bundles with integrable connections or λ -connections. I will show a few theorems concerning moduli spaces of such semistable modules. Some of them are completely new even in the case of usual semistable sheaves and allow, e.g., for a precise description of points of the moduli space of slope semistable sheaves on a variety of any dimension.

Jérémy Toulisse (June 11, 11:45-12:30)

A universal space for maximal representations in rank 2

The universal Teichmüller space, introduced by Bers in 1965, is an infinite space that naturally contains all the Teichmüller space. In the same spirit, we propose a universal space for maximal representations in rank 2, discuss its properties and its relations with self-duality equations on the hyperbolic disk. This is a joint work with François Labourie.

Marina Logares (June 11, 16:30-17:15)

On character varieties of singular manifolds.

The study of the algebro-topological invariants of G -character varieties for a Riemann surface roots, in their relation through the non-abelian hodge correspondence, to the moduli space of G -Higgs bundles. This also inspired the study of the same invariants for G -character varieties of Riemann surfaces without a finite number of points. One way to address this study is to construct a lax TQFT that computes their virtual class in the Grothendieck ring of algebraic varieties. In this talk we want to show that it can also be applied to singular curves such as nodal curves. This is based on joint work with Angel González Prieto.

Richard Wentworth (June 11, 17:15-18:00)

Line bundles on moduli space

In this talk I will discuss an approach to the construction of connections and metric structures on natural line bundles that exist over moduli spaces of flat connections on families of Riemann surfaces. These line bundles are associated with the determinant of cohomology and the (complex) Chern-Simons functional. Applications include a simple description of hyperholomorphic line bundles, curvature computations, and an interpretation of the Cappell-Miller torsion. This is joint work with Dennis Eriksson and Gerard Freixas i Montplet.